Electronic barter market

of fungible values

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abstract

The model openBarter defines an central limit order book allowing cyclic exchanges between more than two partners (buyer and seller) in a single transaction. It does not require any central monetary standard to perform competition between orders. The model provides a liquidity nearly equivalent to that of a regular market when the diversity of qualities is not too large.

A value is defined by a couple **(quantity,quality)**, where quantity is an integer, and quality a name.

An order is as a statement of an owner to provide a value in exchange of an other value. The ratio between provided and required quantities defines a number that can be used by the owner to define the limit of its order as he would do with a limit price on a regular market.

We will consider first the bilateral case in the §1, and then will extend it to wider cycles.

1 Bilateral exchange

Using the regular definition of price for an exchange between a seller providing 10 Kg of apples in exchange of 20 pounds, the price is 2 pounds/Kg. Likewise, if a seller provides a quantity g of goods to a buyer in exchange of a quantity m of money, the regular definition of price is: $p=\frac{m}{g}$.

Buyer and seller have usually different ideas of prices, We note the buyer price p_b and the seller price p_s . An agreement between the buyer and the seller is the result of a barter between these prices.

If we consider now the ratio between quantities provided and received that depends on the view-point.

this ratio is ω_b for the buyer and such as $\omega_b=\frac{m}{g}$

and ω_s for the seller such as $\omega_s=\frac{g}{m}$.

We can observe that $\omega_b=p_b$ and $\omega_s=\frac{1}{p_s}$

1.1 Agreement

Agreement on price between the buyer and the seller exists when their respective prices $\boldsymbol{p_b}$ and $\boldsymbol{p_s}$ are equal: $p_b = p_s$. An equivalent statement using expressions ω_b and ω_s of price is:

[1] $$\omega_b = \frac{1}{\omega_s} \Rightarrow \omega_b * \omega_s = 1$$

1.2 Best matching

Using the regular definition of price, preference of prices depends on the point of view:

- For the buyer, $p_{b1} \geq p_{b2}$ means p_{b2} is better than p_{b1} .
- For the seller, $p_{s1} \leq p_{s2}$ means p_{s2} is better than p_{s1} .

Even these rules are distinct, they are both called the best price rule!

1.3 Compromise

Compromise is required when buyer and seller do not agree on their prices; that is when $p_b \neq p_s$. If we choose the geometric mean of p_b and p_s: $p' = \sqrt{p_b * p_s}$ as the result of the compromise we can observe that $\frac{p_b}{p'} = \frac{p'}{p_s}$. Since an increase for one side should be compared to a decrease for the other these ratios $\frac{p_b}{p'}$ and $\frac{p'}{p_s}$ measure the negotiation effort of the buyer and the seller respectively. Such a rule that balance efforts of partners is the reason why we propose to choose this geometric mean.

Expressed using ω, this barter provides new values ω'_s and ω'_b such as:

$$\omega_s' = \frac{\omega_s}{\sqrt[2]{\omega_s * \omega_b}}$$
$$\omega_b' = \frac{\omega_b}{\sqrt[2]{\omega_s * \omega_b}}$$

We can verify that their product is 1 as expected by [1].

The result of this compromise has the same form for both partners:

[2] $$\omega' = \frac{\omega}{\sqrt[2]{\omega_s * \omega_b}}$$

It is used to compute the quantity of money and of stock finally exchanged between the buyer and the seller.

1.4 Maximum order book

The process performed by a maximum order book when a new order (bid or ask) is submitted can be presented as follows:

A. The list of orders matching this new order is obtained from the order book. If this list is empty, the new order is stored in the order book and the process stops.

B. The best order is found from this list according to the best price rule,

C. The execution produces a couple of movements between partners that reduces the provided and required quantities of the couple of orders. Among these two orders, at least one is complete. This order is removed from the order book. If the new order is not complete, the process is repeated from A. Otherwise the process stops.

The matching can be a “limit price” or “market”. In both cases, the matching is possible when the quality of the stock provided by the bid is the same as the quality required by the ask. For a “limit price”, bids are rejected if the compromise is higher than a limit defined by the buyer, and asks are rejected if the compromise is lower than a limit defined by the seller.

The matching requires a compromise between the couple of orders. This compromise is required to find an agreement between buyer and seller and finally execute their orders. It is a meaning of the word 'barter' different from an economic exchange that do require any currency. We will only use the latter in this paper.

2 Multilateral exchange

2.1 Agreement

Let's consider a list of n orders where the quality offered by one equals the quality required by the following, and where the quality offered by the last equals the quality required by the first order. This list forms a potential multilateral exchange cycle. An order i with $i \in [0, n-1]$ offers a quantity qp_i and requires a quantity qr_i. Let $\vec{\omega}$ be the vector with coordinates ω_i : .

$$\omega_i = \frac{qp_i}{qr_i}$$

Let's suppose an agreement exists between partners, and that each partner i , with $i \in [0, n-1]$, of the cycle provides a given quantity q_i to an other partner. Let $\vec{\omega}'$ be the vector with coordinates ω'_i ratio between quantity provided and quantity received by a partner i :

$$\omega'_i = \frac{q_i}{q_{i-1}} \quad for \quad i \in [1, n-1]$$

$$\omega'_0 = \frac{q_0}{q_{n-1}}$$

If we make the product of those n expression,

we obtain: $$\prod_{i=0}^{n-1} \omega'_i = \frac{q_0}{q_{n-1}} * \prod_{j=1}^{n-1} \frac{q_i}{q_{i-1}} = 1$$

We see that when an agreement exists, we have:

[3] $$\prod_{i=0}^{n-1} \omega'_i = 1$$

We also see that [1] is a special case of [3].

If the product of ω_i is 1, we just choose $\vec{\omega} = \vec{\omega}'$ in order to verify this necessary condition.

Otherwise, $\vec{\omega}$ need to be adjusted to obtain a compromise $\vec{\omega}'$.

2.2 Best matching

When a new order is added to the order book, the search for coincidence provides a set of cycles that will be transformed into agreements using a priority order that should comply with the best price rule of the bilateral case.

A potential agreement is defined by a cycle C of orders and corresponding ω_i with $i \in [0, n-1]$.

For a cycle C, we note Ω the product $\prod_{i=0}^{n-1} \omega_i$.

We propose to define a total order $.\preccurlyeq.$ on the set of cycles found as follows:

[4] $$C_1 \preccurlyeq C_2 \Leftrightarrow \Omega_1 \leq \Omega_2$$

Since this order is total, it defines a maximum cycle on the set of cycles.

Let's consider the best price rule from the view-point of buyers and sellers.

A buyer compares cycles C_1 and C_2 when a common order belongs to both – it is an ask in this case. It's price is p_b, and prices of corresponding bids are p_{s1} and p_{s2}. He chooses C_2 when $p_{s2} \leq p_{s1}$. Since $\omega_s = \frac{1}{p_s}$ we have: $p_{s2} \leq p_{s1} \Rightarrow \omega_{s1} \leq \omega_{s2} \Rightarrow \omega_b * \omega_{s1} \leq \omega_b * \omega_{s2} \Rightarrow C_1 \preccurlyeq C_2$

A seller compares cycles C_1 and C_2 when a common order belongs to both – it is a bid in this case. It's price is p_s, and prices of corresponding asks are p_{b1} and p_{b2}. He chooses C_2 when $p_{b1} \leq p_{b2}$. Since $\omega_b = p_b$ we have: $p_{b1} \leq p_{b2} \Rightarrow \omega_{b1} \leq \omega_{b2} \Rightarrow \omega_{b1} * \omega_s \leq \omega_{b2} * \omega_s \Rightarrow C_1 \preccurlyeq C_2$

Hence, we can state that the order [4] comply with the best price rule of bilateral exchanges.

2.3 Compromise

We propose to extend [2] to a cycle C by choosing ω'_i as :

$$\omega_i' = \omega_i * \left(\frac{1}{\Omega} \right)^{\frac{1}{n}}$$

Since Ω is equally shared between partners, this is a fair compromise when all partners of the cycle of bids are distinct. But several orders on a cycle may belong to the same partner. This can be an opportunity for him to take advantage of the situation. This can never occur on bilateral exchanges since when several orders belong to the same partner, the partner is alone to exchange.

The compromise is hence modified by sharing first Ω between partners, then by sharing the results between orders of each partner. Formally let *m* be the number of partners, such as $m \leq n$ and let o_i be the number of orders of the author of the order i , such as $\sum_{i=0}^{m-1} o_i = n$. The compromise is:

$$\omega_i' = \omega_i * \left(\left[\frac{1}{\Omega} \right]^{\frac{1}{m}} \right)^{\frac{1}{o_i}}$$

or :

[5] $$\omega_i' = \omega_i * \Omega^{-\frac{1}{m*o_i}}$$

that verify the equality [3] .

This compromise remains fair even if more than one order belongs to the same partner in the cycle.

Once ω_i' are obtained from [5], the next step is to find a vector $\vec{Q}$ of integers representing quantities to be exchanged by the draft agreement from quantities offered by orders.

A vector $\vec{q}$ of real numbers is first computed as the maximum flow circulation through the cycle with the constraints of ω_i' and of quantities offered by orders (see annexe I). Rounding of $\vec{q}$ to obtain $\vec{Q}$ has several solutions since each coordinate can be rounded to lower or upper bound. We exclude from them those where some coordinate of $\vec{Q}$ is null because a draft agreement where some partners don't provide anything would be unfair. Among remaining candidates, we choose the one minimizing a distance between the two vectors $\vec{q}$ and $\vec{Q}$. This distance is chosen as the angle between the two vectors.

This computation finally provides $\vec{Q}$ representing quantities of the draft agreement defining a fair compromise between orders of the cycle.

2.4 Maximum order book

The process performed by this maximum order book when a new order is submitted is very similar to a bilateral exchange:

A. The list of cycles matching the new order is obtained from the order book. Each cycle is itself a list of orders including the new order. If this list is empty, the new order is stored in the order book and the process stops.
B. The best cycle is found from this list according to §2.2,
C. A compromise is found between orders of this best cycle according to §2.3 to find an agreement between partners and finally execute these orders. The execution produces a set of movements between partners that reduces the provided quantities of the orders of the cycle. Among these orders, at least one is complete. If the new order is not complete, the process is repeated from A. Otherwise the process stops.

The matching can be a “limit $\boldsymbol{\omega}$” or “market”. In both cases, the matching is possible when the quality of the stock provided by an order is the same as the quality required by the following. For a “limit $\boldsymbol{\omega}$”, orders are rejected if the compromise is higher than a limit defined by the partner.

3 Implementation of the model

This is by far the most time-consuming part of the work for the following reasons:

- Integrity of data is the primary concern of users of this maximum order book. Hence, the whole insertion process of a single order must be wrapped in a single atomic transaction that can be rolled back in case of hardware failure.
- Security of external access to the maximum order book is an other primary requirement.
- Graph algorithms that explores a large amount of data are far more time-consuming than regular maximum order books that provide a result in a single millisecond.

This is the reasons why the following work were made:

- Use of a tool that provides security and integrity that can be easily customized. PostgreSQL is good candidate among existing databases due to its maturity, its extensibility, and to the quality and size of the worldwide community that developed it.
- Development of an adaptation of the Bellman-Ford algorithm to find cycles with Ω maximum. This algorithm required the graph to be acyclic.
- Reduce the graph exploration to cycles having a limited number of nodes, while maintaining the acyclic constraint for those cycles.
- Adaptation of this algorithm that tolerates cycles.
- Integration of this algorithm in postgreSQL while maintaining its original security and integrity properties.
- Sharing of this algorithm between standards primitives of postgreSQL and fast subroutines using the C native language and extensibility mechanisms of the database.

The result of this work has been published under open-source licence:

http://olivierch.github.com/openBarter

It provides a set of services available through the standard interface of postgreSQL whose main primitives are the followings:

- Read the market price (ω) for a given couple (quality provided, quality required),
- Get a quote by providing (owner, quality and quantity provided, quality and quantity required),
- Insert an order defined by (owner, quality and quantity provided, quality and quantity required).

The results of the process are movements that swap round the property of stocks made available by orders.

3.1 Benchmark

Order has been submitted continuously for 10 hours 47 minutes. 3405 orders was inserted; the mean time of execution was 11,4 seconds.
599 exchange cycles was produced described by 1986 account movements.
The illustration 1 represents of the 925 active orders remaining at the end of activity. Nodes represent 20 distinct qualities exchanged, and directed arcs the

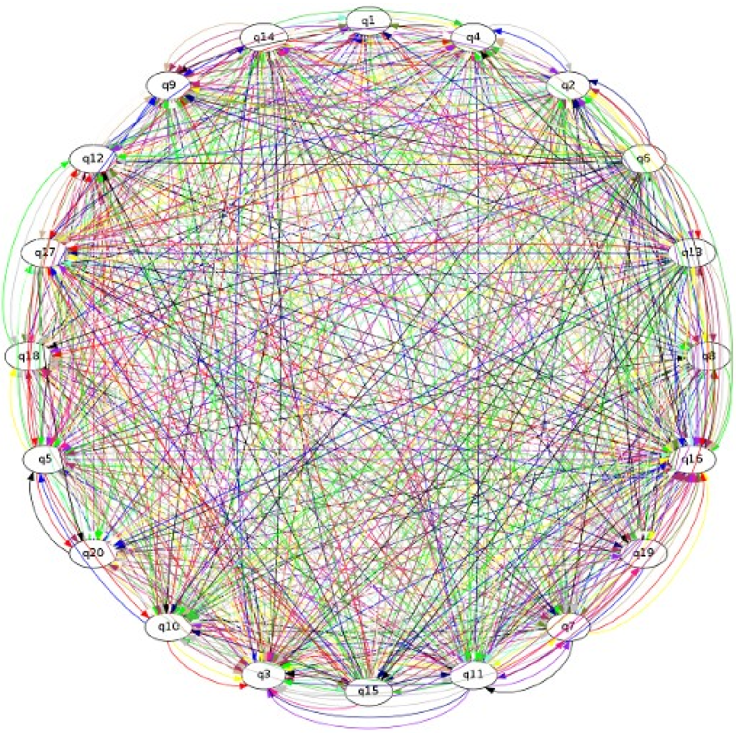

Illustration 1: graph representation of the order book

925 orders. Their colours represent the author of the order also owner of the quantity offered.

599 exchange cycles was produced, described by 1986 account movements. Cycles contained at most 8 movements, The distribution of the length of cycles presented on illustration 2. Not surprisingly it shows that cycles remain short when the diversity of qualities is low.

number of exchanges	**cycles**
2	203
3	175
4	110
5	59
6	45
7	6
8	1
Total	**599**

Illustration 2: distribution of the length of cycles

If the maximum length of cycles was reduced to 6 it would not decrease a lot the chance of matching and increase performance.

The study of the distribution of the length of cycles in a relation with the number of orders contained by the order book and the variety of qualities could bring an interesting response to such optimisation.

The experience showed that unwanted orders tend to decrease performance when the book grows, and that exhaustive exploration of the order book was not a

The software has been improved in order to increase performance:

- The life time of orders in the book was limited. The life time measurement was chosen in order not to penalize orders containing uncommon qualities.
- The maximum number of path extracted from the graph used to build the cycle candidates was limited to 100 000.

A new benchmark was performed on this new software with 10 000 orders. The mean time of execution was 3 seconds, this time including all required integrity and security mechanisms offered by PostgreSQL.

The most recent benchmark using the PostgreSql v9.2 gave 0.3 second by order for 100 000 orders.

4 Integration into existing infrastructure

The model can be seen as a black box with a flow of orders as input and a flow of movements as output. To be used, the database implementing the model need to be connected to institutions recording the ownership of values. This connection could be implemented by extending the FIX protocol used by major financial institutions to exchange trade related messages.

A front office should be developed, presenting to owners an history of last transactions, best opportunities of the market, and an interface to submit or cancel orders.

5 Utility of the model

Day-to-day experience shows that the difficulty of driving a system grows as its complexity. Using the words of cybernetics the number of dimensions of a command signal required to maintain the stability of a system increases as its complexity. Applied to the stability of life on earth and if we consider the dollar as a mono-dimensional command signal, it is not surprising that many attempts to adapt the markets to the urgent needs of ecological economics is not convincing so far.

The physical model of greenhouse gases for example depends on a diversity of gas spices including many trace gases whose abundances need to be accurately monitored[1]. The primary requirement of adaptation is the multidimensionality of resources. Unfortunately, the urgency of building of a market linking this model to the financial systems imposed a simplification of its command signal to a single dimension, the carbon.

But it is the physical reality of natural processes that should drive adaptation of markets and not the contrary. This model openBarter provides a mean to adapt allocation of resources of the market economy to the reality of natural processes.

1 W.C. WANG and others, 'Greenhouse Effects Dues to Man-Made Perturbations of Traces Gases', *SCIENCES*, 194 (1976), 685,690.

ANNEXE I – Maximum flow

From $\vec{\omega}$ and quantities offered by orders we describe how the maximum flow $\vec{q}$ is obtained.

If integers a_i with $i \in [0, n-1]$ define the quantities of orders then the flow $\vec{q}$ should verify the following constraints:

$$\frac{q_i}{q_{i-1}} = \omega_i \quad for \quad i \in [1, n-1]$$
$$\frac{q_0}{q_{n-1}} = \omega_0 \quad for \quad i = 0$$
$$q_i \leq a_i \quad for \quad i \in [0, n-1]$$

To find this $\vec{q}$ let's first consider the ratios $\frac{q_i}{q_0} = \pi_i$. They are such as:

$$\pi_0 = 1$$
$$\pi_i = \prod_{j=0}^{i-1} \omega_j \quad for \quad i \in [1, n-1]$$

We can observe that: $q_i \leq a_i \Rightarrow \frac{q_i}{\pi_i} \leq \frac{a_i}{\pi_i} \Rightarrow q_0 \leq \frac{a_i}{\pi_i}$

and that:

$$q_0 \leq \frac{a_i}{\pi_i} \; for \; i \in [0, n-1]$$

If we choose $\vec{q}$ such as $q_0 = min(\frac{a_i}{\pi_i})$ and then $q_i = q_0 * \pi_i$, we see that $\vec{q}$ verify all constraints.